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METEOROLOGICAL RESEARCH COMMITTEE

The relationship between standard deviation of contour height and standard vector deviation of wind, with practical applications

by

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Summary

The paper is in two parts. In the first part a formula is derived, believed to be valid everywhere outside the tropics and at all levels between the top of the friction layer and about 300 mb., expressing standard vector deviation of wind as a universal constant multiplied by the product of the standard deviation of contour height and the cosecant of latitude.

The formula may be written $\sigma = 0.064\, s \operatorname{cosec} \phi$ where σ is the standard vector deviation of wind in kt., s is the standard deviation of contour height in ft. and ϕ is the latitude.

A similar relationship is derived for levels between 300 mb. and 100 mb. We may write $\sigma = ks \operatorname{cosec} \phi$ where k is constant for different places but decreases uniformly with height from 0.064 at 300 mb. to 0.033 at 100 mb.

In the second part a formula is derived expressing the standard deviation of contour height, at levels up to about 300 mb., in terms of the standard deviation of surface pressure and the mean standard deviation of temperature.

The relationship $\sigma = 0.064\, s \operatorname{cosec} \phi$ was then used to compute the standard vector deviation of wind at 300 mb. over the northern hemisphere during the winter season, December to February.

World seasonal maps of the standard vector deviation of wind above the friction layer have been prepared from corresponding maps of the standard deviation of surface pressure; and seasonal maps of standard vector deviation of wind at 500 mb. over part of the North Atlantic were prepared from maps of standard deviation of contour height computed directly from daily values.

Some discussion is given of the variation with height of standard vector deviation of wind.

1. The relationship between standard deviation of contour height and standard vector deviation of wind.

(a) Derivation of the relationship

Let P_1 and P_2 be two points close together on an isobaric surface. Let y be the distance between the points and h_1 and h_2 the contour heights of P_1 and P_2. Then if u is the mean component

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of geostrophic wind speed in the direction perpendicular to the line P_1P_2

$$\lambda yu = h_1 - h_2 \qquad (1)$$

where $\lambda = 2(\omega/g)\sin\phi$

ω is the earth's angular velocity, g is the acceleration due to gravity and ϕ is latitude.

Then $$s_{12}^2 = s_1^2 - 2r_{12}s_1s_2 + s_2^2 = \lambda^2y^2\sigma_u^2 \qquad (2)$$

where s_{12} is the standard deviation of the difference, $(h_1 - h_2)$, in contour heights, s_1 and s_2 are standard deviations of contour height at P_1 and P_2, r_{12} is the correlation coefficient between h_1 and h_2, and σ_u is the standard deviation of the velocities u.

Equation (2) may be re-arranged

$$s_{12}^2 = (s_1 - s_2)^2 + 2s_1s_2(1 - r_{12}) = \lambda^2y^2\sigma_u^2 \qquad (3)$$

In the limit, as $y \to 0$
$r_{12} \to 1$ $\qquad s_1 - s_2 \longrightarrow 0$

So it is not obvious what are the relative contributions of $(s_1 - s_2)^2$ and $2s_1s_2(1 - r_{12})$ to their sum s_{12}^2.

But consideration of actual examples shows that

$$(s_1 - s_2)^2 \ll 2s_1s_2(1 - r_{12})$$

The values of

$$s_{12}^2,\ s_1,\ s_2 \quad \text{and} \quad (s_1 - s_2)^2$$

were computed and are given in Table 1 for the point pairs

(i) Larkhill and Liverpool at 200 mb. for January, 1952.

(ii) Habbaniya and Bahrein at 300 mb. for January, 1950-52.

(iii) Malta and Benina at 300 mb. for January, 1950-52.

(iv) Tateno and Kagoshima at 200 mb. for October, 1951.

(v) Tateno and Kagoshima at 200 mb. for July, 1950-51.

TABLE 1. CORRESPONDING VALUES OF s_1, s_2, $(s_1 - s_2)^2$, s_{12}^2

	(i)	(ii)	(iii)	(iv)	(v)
Units of height	Tens of feet			metres	
s_1	66	31	43	100	90
s_2	68	29	33	76	59
$(s_1 - s_2)^2$	4	4	100	576	961
s_{12}^2	657	1444	1529	5853	10920

Authorities for data:- (i), (ii), (iii), MS. data available in the Upper Air Branch of the Meteorological Office; (iv) and (v), Tokyo, Aerological Data of Japan.

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In equation (3) divide throughout by y^2 and take limits as $y \longrightarrow 0$, when P_1 and P_2 approach the point P.

Then $s_1 \rightarrow s$ $s_2 \rightarrow s$ $(s_1 - s_2)^2/y^2 \longrightarrow (\partial s/\partial y)^2$ where s is the standard deviation of contour height at P.

Since in the examples quoted in Table 1 $(s_1 - s_2)^2$ makes only a small contribution to the value of s_{12}^2, it may reasonably be deduced that

$$\lim_{y \rightarrow 0} \frac{2s_1 s_2 (1 - r_{12})}{y^2}$$

exists and is very nearly equal to $\lambda^2 \sigma_u^2$. That is

$$\sigma_u^2 = 2s^2/\lambda^2 \lim_{y \rightarrow 0} (1 - r_{12})/y^2 \qquad (4)$$

Equation (4) at once arouses speculation about

$$\lim_{y \rightarrow 0} (1 - r_{12})/y^2$$

If the limit is the same for any direction of P_1P_2 then σ_u^2 is the same for all directions, since λ and s are dependent only on the position of P. That is, the wind distribution is circular, and

$$\sigma_u^2 = \sigma^2/2 \qquad (5)$$

where σ is the standard vector deviation of wind.

Conversely, if the wind distribution is circular, then

$\lim_{y \rightarrow 0} (1 - r_{12})/y^2$ is independent of direction.

It has been shown by Brooks and others (1950) that wind distributions are very nearly circular. So it follows that

$$\lim_{y \rightarrow 0} (1 - r_{12})/y^2$$

is the same for all directions.

It may then be asked whether the limit is the same for all levels, and whether indeed it is a universal constant. If the limit were a universal constant then equation (4), modified by equation (5), would take the form

$$\sigma = ks \operatorname{cosec} \phi \qquad (6)$$

where k is a universal constant, the same for all places and all levels.

(b) Verification of the relationship for levels up to 300 mb.

The validity of the hypothesis expressed in equation (6) has been tested on corresponding data of standard deviation of contour height and standard vector deviation of wind for eleven stations in the northern hemisphere. The data are given in Table 2. It should be noted that the validity of the hypothesis cannot be tested for stations in the tropics for two reasons: (i) the standard deviation of error in measurement of contour height may be of the same order as the true standard deviation of contour height, and hence the standard deviation of contour height given by the data may be altogether incorrect, (ii) the derivation of equation (6) depends on the geostrophic relationship, which does not apply near the equator.

/TABLE 2.

TABLE 2. CORRESPONDING VALUES OF STANDARD DEVIATION OF CONTOUR HEIGHT (FT./10) AND STANDARD VECTOR DEVIATION OF WIND (KT.)

STATION NAME LATITUDE PERIOD	MONTHS	850mb. s	850mb. σ	700mb. s	700mb. σ	500mb. s	500mb. σ	300mb. s	300mb. σ	200mb. s	200mb. σ	150mb. s	150mb. σ	100mb. s	100mb. σ	Authority
LERWICK * 60°8' 1948-50	Dec. - Feb.	44	29	47	29	57	38	77	51	72	38	68	29	70	26	
	Mar. - May	35	24	41	26	53	35	69	47	59	33	57	25	60	21	
	June - Aug.	23	19	27	20	36	28	53	42	49	31	40	19	36	12	a
	Sept. - Nov.	36	26	38	26	47	35	65	52	61	39	53	27	47	18	
LARKHILL * 51°12' 1948-50	Dec. - Feb.	35	29	40	32	53	42	69	56	65	43	56	31	49	25	
	Mar. - May	28	22	33	26	43	34	72	49	50	36	43	24	42	18	
	June - Aug.	21	17	25	21	34	29	50	45	49	39	41	26	33	15	a
	Sept. - Nov.	27	22	32	24	44	34	61	49	62	43	54	31	42	19	
GIBRALTAR 36°9' 1950-52	Jan.			26	23	35	32	48	45	53	39	50	26	44	20	
	Apr.			16	19	20	28	30	39	31	33	25	22	27	18	
	July			13	15	16	21	23	29	28	33	30	26	27	19	b
	Oct.			15	16	22	20	32	35	34	34	40	27	28	16	
MALTA 35°53' 1950-52	Jan.	19	19	23	21	30	31	43	48	39	37	39	29	31	22	
	Apr.	15	18	18	18	25	26	38	36	35	32	30	23	29	18	
	July	7	11	8	16	11	18	20	22	29	27	33	22	23	15	b
	Oct.	14	14	17	17	23	24	27	34	31	39	30	28	33	18	
NICOSIA 35°12' 1950-52	Jan.			18	18	28	29	39	43	39	39	33	26	31	21	
	Apr.			20	17	29	24	42	37	41	30	40	24	42	17	
	July			8	9	11	16	21	22	24	24	26	21	28	18	b
	Oct.			10	15	14	18	24	27	30	28	33	26	31	16	
HABBANIYA 33°22' 1950-52	Jan.			16	15	24	29	31	40	37	29	37	29	32	17	
	Apr.			14	17	24	26	35	34	40	34	38	29	35	19	
	July			10	14	13	16	16	19	19	21	24	20	29	15	b
	Oct.			9	14	17	15	21	25	28	30	28	24	26	17	
BENINA 32°6' 1950-52	Jan.			15	20	23	28	33	43	36	39	35	24	33	23	
	Apr.			17	16	23	25	30	35	37	33	34	27	29	18	
	July			9	14	10	14	18	22	24	23	27	22	31	15	b
	Oct.			11	18	13	19	22	27	29	29	30	24	29	15	
BAHREIN 26°14' 1950-52	Jan.			13	16	18	23	29	36	30	33	29	22	29	18	
	Apr.			9	15	13	20	22	30	26	38	27	31	25	21	
	July			8	15	9	13	11	14	16	14	20	15	27	15	b
	Oct.			7	13	9	14	18	20	19	21	21	19	26	18	
WAKKANAI 45°25' 1952	Jan. - Feb.	18	14	20	19	27	27									
	July - Aug.	15	13	18	15	14	19									c
TATENO 36°3' 1950-52	Jan.			23	22	32	34	42	48							
	Apr.			21	25	32	40	47	48							
	July			16	15	19	24	24	34							c
	Oct.			19	18	26	27	35	38							
MARCUS I 24°18' 1952	Jan. - Feb.			9	15	10	20	14	27							
	July - Aug.			7	12	8	15	11	17							c

NOTE * Values of s and σ are mean monthly values for each season.

AUTHORITIES (a) Published values of s and σ ; Meteorological Office, M.O.555, Parts 1, 2 and 3.

(b) Values of s and σ were computed from manuscript tabulations of contour height and upper wind available in the Upper Air Branch of the Meteorological Office.

(c) Values of s and σ were computed from published tabulations of contour height and upper winds; Tokyo, Aerological Data of Japan.

Combining together data from all stations, a separate graph for each of the levels 850, 700, 500, 300, 200, 150 and 100 mb. was drawn with σ plotted against s cosec ϕ . These are shown in Fig. 1 (a, b, c, d, e, f, g). The units of speed are kt., and of contour height, ft.

(i) The data for 850, 700, 500 and 300 mb.

Very good fits for the data at each of the levels 850, 700, 500 and 300 mb. are provided by straight lines passing through the origin. These are shown in Fig. 1 (a, b, c, d). The mean error in σ at 300 mb. is only eight per cent., that is $1\frac{1}{2}$ kt. for σ = 20 kt. and 4 kt. for σ = 50 kt. The best fitting straight lines for the individual levels 850, 700, 500 and 300 mb. are:-

850 mb. $\sigma = 0.062$ s cosec ϕ

700 mb. $\sigma = 0.063$ s cosec ϕ

500 mb. $\sigma = 0.065$ s cosec ϕ

300 mb. $\sigma = 0.064$ s cosec ϕ

It will thus be seen that the straight line

$$\sigma = 0.064 \text{ s cosec } \phi$$

gives an excellent fit to corresponding values of σ and s at each of the levels 850, 700, 500 and 300 mb., showing that the relationship between σ and s is well expressed by equation (6), with k = 0.064, for widely different latitudes and at all levels from 850 to 300 mb. The straight line $\sigma = 0.064$ s cosec ϕ also provides an excellent fit for each station separately, with data for the levels 850, 700, 500 and 300 mb. combined.

The data show some slight seasonal variation in the value of k, with k appearing to have a rather higher value in summer than in the other seasons; but this may be partly accounted for by errors in the summer standard deviations of contour height. The variation is least at 300 mb.

The seasonal variations in k; (1) for each station separately, with all levels taken together; (2) for each level separately, with all stations taken together; (3) for all stations combined and all levels taken together, are shown in Table 3.

/TABLE 3.

TABLE 3. SEASONAL AND ANNUAL VARIATIONS IN THE VALUE OF k FOR THE LEVELS OF 850, 700, 500, 300 MB.

		Winter	Spring	Summer	Autumn	Year
(1)	Lerwick	.057	.058	.065	.065	.061
	Larkhill	.063	.064	.066	.062	.064
	Gibraltar	.056	.077	.074	.061	.062
	Malta	.062	.060	.085	.065	.065
	Nicosia	.062	.050	.068	.064	.062
	Habbaniya	.066	.059	.070	.064	.066
	Benina	.068	.062	.071	.076	.068
	Bahrein	.056	.066	.067	.062	.059
	Wakkanai	.065		.070		.068
	Tateno	.064	.066	.073	.062	.065
	Marcus I	.077		.070		.074
(3)		.062	.062	.070	.064	.064
(2)	850 mb.	.060	.059	.070	.062	.062
	700 mb.	.059	.061	.070	.066	.063
	500 mb.	.062	.063	.073	.063	.065
	300 mb.	.064	.062	.068	.065	.064

(1) Each station separately, all levels combined.
(2) All stations combined, each level separately.
(3) All stations combined, all levels combined.

(c) The form of the relationship for levels above 300 mb.

(i) The data for 200, 150 and 100 mb.

Fig. 1 (e, f, g) shows the data for each level separately, with the best fitting straight lines drawn. The fits are good, and for each level separately a relationship between σ and s

$$\sigma = ks \operatorname{cosec} \phi$$

holds good, but with different values for k at each level, and different also from the value k = 0.064 which holds good for every one of the levels 850, 700, 500 and 300 mb.

The best-fitting straight lines for the 200, 150 and 100 mb. levels give values for k equal to 0.054, 0.043 and 0.033 respectively. The seasonal and annual variation of k at each level is shown by the data of Table 4.

TABLE 4. SEASONAL AND ANNUAL VARIATIONS IN THE VALUE OF k FOR THE LEVELS OF 200, 150, 100 MB.

		200 mb.	150 mb.	100 mb.
(1)	Winter	.050	.039	.034
	Spring	.053	.043	.033
	Summer	.057	.044	.032
	Autumn	.057	.045	.033
	Year	.054	.043	.033
(2)	Lerwick	.051	.040	.032
	Larkhill	.056	.045	.036
	Gibraltar	.056	.041	.033
	Malta	.060	.045	.037
	Nicosia	.052	.044	.032
	Habbaniya	.051	.044	.031
	Benina	.053	.041	.031
	Bahrein	.051	.040	.030

(1) Seasonal and annual values, all stations combined.
(2) Annual values, each station separately.

/(ii)

(ii) Variation of k with height

The best fitting values of k were plotted in Fig. 2 as a graph of height (logarithm of pressure) against k.

The variation of k with height is seen to be represented very well by two straight lines, k constant and equal to 0.064 up to 300 mb., and k decreasing thereafter uniformly with height to a value of 0.033 at 100 mb.

Brooks and others (1950) have shown that standard vector deviation usually increases up to a certain level, above which it decreases. This level of discontinuity is associated with the tropopause, but because of variations in tropopause height it is usually below the average level of the tropopause.

The level of 300 mb. may be considered as the mean level of discontinuity for the stations examined.

(d) Conclusion

It may be concluded that at all levels from the top of the friction layer to about 300 mb. there is a universal relationship between the standard vector deviation of wind and the standard deviation of contour height which may be written in the form of equation (6)

$$\sigma = 0.064 \, s \, \mathrm{cosec} \, \phi \, .$$

It may also be asserted that above 300 mb. a similar relationship is approximately true, $\sigma = ks \, \mathrm{cosec} \, \phi$, with k now varying with height and taking values which lie on the straight line of Fig. 2, that is,

$$k = 0.064 - 0.028 \ln (300/p)$$

where p is the pressure in mb.

2. Some practical applications.

(a) Introduction

Sufficient wind data are not available to enable maps of standard vector deviation of wind to be drawn for many areas.

The results obtained in part one may be used to estimate standard vector deviation of wind from standard deviation of contour height. Maps of standard deviation of contour height can be prepared from daily values of contour height read from working charts. This procedure is however rather laborious, and an alternative method of deriving standard deviation of contour height at levels up to 300 mb. is given in the following section.

/(b)

(b) Preliminary - an expression for the standard deviation of contour height at levels up to 300 mb.

If h is the contour height of the pressure surface p

h_o is the height of the 1000 mb. surface

h' is the height from 1000 mb. to pressure p

R is the gas constant

θ is the mean (log p, T) temperature of the column 1000 mb. to p

T_o is the temperature at 1000 mb.

p_o is the pressure in mb. at mean sea level

$\Delta p_o = p_o - 1000$

Then $h = h' + h_o$ and

$$\left.\begin{aligned} h' &= (R/g)\theta \ln(1000/p) \\ h_o &= (R/g)T_o \ln(p_o/1000) = (R/g)T_o(\Delta p_o/1000) \end{aligned}\right\} \quad (7)$$

neglecting in the second of these equations terms of order $(\Delta p_o/1000)^2$.

Now $$s^2 = (s')^2 + 2rs's_\theta + s_o^2 \quad (8)$$

where s' and s_o are the standard deviations of h' and h_o respectively; and r is the correlation coefficient between h' and h_o. r is small, and is found to have a value of very nearly 0.35.

From equation (7) we derive

$$s' = (R/g)\sigma_\theta \ln(1000/p) \quad (9)$$

where σ_θ is the standard deviation of the mean temperatures θ. Also, since the proportional variation of T_o is very small compared with that of Δp_o we can write, with sufficient accuracy,

$$s_o = (R/g)\bar{T}_o(\sigma_p/1000) \quad (10)$$

where $\bar{T}_o$ is the average temperature at 1000 mb. and σ_p is the standard deviation (mb.) of surface pressure.

Consideration of actual examples shows that the error introduced by ignoring the variation of T_o is about one per cent., and this is less than the error in the computed values of h_o.

If we use the notation $\sigma_T(\pi)$ for the standard deviation of temperature at fixed pressure levels π between 1000 mb. and p, and write σ_T for the root mean square of the quantities $\sigma_T(\pi)$; then by the usual algebra of analysis of variance it can be shown that σ_θ is less than σ_T.

For pressure levels p above about 700 mb., an examination of data for the summer and winter seasons for Larkhill and Malta shows the value of the ratio σ_θ/σ_T to be nearly 0.85. Substituting this value for the ratio in equation (9) we have

$$\begin{aligned} s' &= 0.85(R/g)\ln(1000/p)\,\sigma_T \\ &= 0.85\,s'' \qquad \text{defining } s'' \end{aligned} \quad (11)$$

Substituting the known value 0.35 for r in equation (8)

$$s^2 = (s')^2 + 0.7s's_o + s_o^2 \quad (12)$$

that is, $$s^2 = 0.72(s'')^2 + 0.6s''s_o + s_o^2 \quad (13)$$

Equation (13) was used to compute s at 300 mb. for several places in different seasons. These computed values are compared with actual values in Table 5.

/TABLE 5.

TABLE 5. MEANED MONTHLY VALUES OF σ_T, s_o, s_{300}

		σ_T (°C)	s_o	s_{300} ft./10	computed s_{300}
Lerwick 1946-50	Dec. - Feb.	5.2	44	77	78
	Mar. - May	5.1	35	69	71
	June - Aug.	3.9	23	53	51
	Sept. - Nov.	4.9	36	65	69
Larkhill 1946-50	Dec. - Feb.	5.1	35	69	71
	Mar. - May	4.4	26	56	58
	June - Aug.	3.8	20	50	48
	Sept. - Nov.	4.3	27	61	58
Malta 1950-52	Jan.	3.4	20	43	44
	Apr.	3.0	17	38	39
	July	2.0	8	20	24
	Oct.	2.3	13	27	30
Habbaniya 1948-50	Dec. - Feb.	3.6	13	38	42
	Mar. - May	3.6	12	37	41
	June - Aug.	1.9 *	8	20	23
	Sept. - Nov.	2.4	8	24	27

Note * July, 1951-52.

Authorities - Malta, and σ_T for Habbaniya, July, 1951-52: computed from MS. data available in the Upper Air Branch of the Meteorological Office; Lerwick, Larkhill, Habbaniya: Meteorological Office, M.O.555, Parts 1, 2 and 3.

The computed values are very good estimates and it may be concluded that the expression for s in equation (13) gives good estimates for the standard deviation of contour height at levels up to about 300 mb.

(c) Estimation of standard vector deviation of the wind above the friction layer

Equation (6)

$$\sigma = 0.064\ s\ \mathrm{cosec}\ \phi$$

can be used to give σ_o, the standard vector deviation of the wind above the friction layer, by substituting s_o for s.

$$\sigma_o = 0.064\ s_o\ \mathrm{cosec}\ \phi \qquad (14)$$

or, combining this equation with equation (10),

$$\sigma_o = 0.00614\ \overline{T}_o\ \sigma_p\ \mathrm{cosec}\ \phi \qquad (15)$$

where the units are

σ_o in kt., $\overline{T}_o$ in °K., σ_p in mb.

Brockmöller (1911) has published data of average monthly pressure range for many stations all over the world. Schumann and Rooy (1951) have published monthly values for the northern hemisphere of standard deviation of daily pressure at points of a grid of latitude and longitude. The values were computed from pressure readings derived from the daily maps of the United States Weather Bureau, analysed once daily, for the period 1929-38.

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The Weather Bureau, South Africa, has published (1952-53) in Notos daily values of mean sea level pressure for the southern hemisphere at points of a grid of latitude and longitude for the period January, 1951 to August, 1952.

Brockmöller's data may readily be converted to monthly standard deviation of daily pressure by dividing by the factor 3.7. The mean range in samples of 30 from a normal population is 4.09 times the standard deviation (Tippett, 1925), but a test with data supplied by the Marine Branch of the Meteorological Office for the weather ships Jig and Item, and with data for stations in Europe, the Mediterranean and Australia gave dividing factors with little variation about a mean of 3.7.

From similar considerations the Notos data were used to give mean seasonal values of standard deviation of daily pressure for the southern hemisphere by dividing mean seasonal values of 10-day pressure ranges by the factor 2.7.

Brockmöller's converted data, the converted Notos data, and the data of Schumann and Rooy were used in equation (15) to give a very large number of values of σ_o for all four seasons over the northern hemisphere between 15°N. and 70°N. and the southern hemisphere between 15°S. and 55°S.

In the tropics use was made of unpublished seasonal maps of the standard vector deviation of surface wind over the oceans, prepared by Mrs. A.H.R. Goldie (Miss N. Carruthers) from data used by the Marine Branch in the construction of charts of surface wind. Since over the oceans the surface wind is approximately two thirds of the wind above the friction layer, the standard vector deviation of the surface wind over the tropical oceans was taken to be approximately two thirds of the standard vector deviation of the wind above the friction layer, with corresponding sets of isopleths forming geometrically similar patterns. This approximate relationship held good in extra-tropical ocean areas, and its use would be expected to give very little error in the tropics, where σ_o is relatively low.

World seasonal maps of σ_o are reproduced in Figs. 3 to 6. The isopleths are given at intervals of 4 kt., but intermediate isopleths, or parts of intermediate isopleths, are drawn as pecked lines in areas where interpolation is thereby made easier.

(d) Estimation of standard vector deviation of wind at 300 mb.

Equations (6) and (13) may be used to give the standard vector deviation of wind at levels up to 300 mb.

Combining them

$$\sigma = 0.064\ (0.72 s''^2 + 0.6 s'' s_o + s_o^2)^{1/2} \operatorname{cosec} \phi \qquad (16)$$

or $$\sigma = (\sigma_o^2 + 0.6 \sigma_o \sigma'' + 0.72 \sigma''^2)^{1/2} \qquad (17)$$

writing σ'' for $0.064\ s''\ \operatorname{cosec} \phi$.

Equation (17) has been used to compute standard vector deviations of wind at 300 mb. in January over the northern hemisphere.

The simplification was introduced of using the standard deviation of temperature at 500 mb., σ_T (500), instead of σ_T , after checking in several areas that this was justifiable during this season. World maps of standard deviations of temperature have been prepared in the Upper Air Branch of the Meteorological Office; and the map for January at 500 mb. was put at my disposal.

/Substituting

Substituting numerical values in the expression for σ'' we obtain

$$\sigma'' = 7.29\ \sigma_T\ \text{cosec}\ \phi \qquad (18)$$

where σ, σ_o, σ'' are in kt. and σ_T or its approximate equivalent $\sigma_T(500)$ is in °C.

From isopleths of $\sigma_T(500)$ values were read off at 30°N., 40°N., 50°N., 60°N. at intervals of 20° longitude. For the points at 30°N. and 40°N. the values of $7.29\ \sigma_T\ (500)\ \text{cosec}\ \phi$ were computed; for 50°N. and 60°N. the values of standard vector deviation at 300 mb. were estimated to have the same value as at 340 mb., since standard vector deviation reaches its maximum value below the tropopause. (Brooks and others, 1950). So at 50°N. and 60°N. values of $6.6\sigma_T(500)\ \text{cosec}\ \phi$ were computed.

At the corresponding points 30° (10°) 60°N., 180°W. (20°) 180°W., values of σ_o were read off. From equation (17) the values of σ_{300}, the standard vector deviation at 300 mb., were computed for the 72 points of this grid. They were plotted, and isopleths of standard vector deviation were drawn. They are shown in Fig. 7.

Some standard vector deviations computed along with vector means for a previous work (Jenkinson, 1954) were added (figures in circles), and standard vector deviations computed from cirrus cloud observations (figures in squares). These were used to draw isopleths south of 30°N.

(e) Estimation of standard deviation of wind at levels above 300 mb.

There appears to be no simple way of deriving standard deviation of contour height, at levels above about 300 mb., from data of standard deviation of surface pressure and standard deviation of temperature at given levels.

And so to use the relationship derived in the first part to estimate standard vector deviation of wind at say 100 mb. we need actual values of s, the standard deviation of contour height, to insert in the formula

$$\sigma = 0.033\ s\ \text{cosec}\ \phi \qquad (19)$$

(k = 0.033 for the 100 mb. level)

Working charts of contours at this level could be used to obtain frequency distributions of contour height at a network of points; and the standard deviation of contour height could then be computed for this network, making allowance for the standard error in contour heights.

This method could of course be used to derive values of s at levels up to 300 mb. It may perhaps be more accurate to use values of s obtained directly in this way than indirectly from equation (13); but the time taken is very much longer.

(f) An example of the mapping of σ at 500 mb. from maps of s prepared directly from daily values of contour height

As an example of the direct transformation of maps of s to maps of σ., average seasonal maps of standard vector deviation of wind at 500 mb. over the North Atlantic (35-65°N., 70°W.-10°E.) were prepared from the corresponding maps of s. From the 500 mb. working charts of the Central Forecasting Office of the Meteorological Office values of contour height were read off to the nearest hundred feet. Charts for January, April, July and October at 1400 G.M.T.

/for

for the period 1949-53 were used, and readings were made at points of the network 35°N. (5°) 65°N., 80°W. (10°) 30°E.

At a selection of points values of s were computed directly. At the other points of the grid values of s were computed from values of the average ten-day range. The formula $\sigma = 0.064\ s\ \mathrm{cosec}\ \phi$ was then used to compute σ at the grid points.

The maps of standard vector deviation of wind are shown in Fig. 8.

(g) Variation with height of standard vector deviation of wind

Clayton (1896), in his discussion of the cloud observations at Blue Hill, said "The velocity of circulation per unit mass is approximately the same at all heights ..... This is what would be expected provided the same mass of air at each cloud level flows through the same vertical section".

The result was confirmed by other writers who worked with the data obtained during the International Cloud Year 1896-97. Egnell (1903) concluded "The mass of air displaced by the wind is constant at all heights. The mean (scalar) speed varies inversely with air density".

Brooks and others (1950) have described the relationship between scalar and vector wind speeds and standard vector deviation, and have enunciated the rule that the product of standard vector deviation with density is constant with height up to a level somewhat below the tropopause; and have suggested that in the tropics the rule holds for levels up to 200 mb. They give the ratios

$$\sigma_{500}/\sigma_{700} = 1.32$$

$$\sigma_{300}/\sigma_{700} = 2.0$$

$$\sigma_{200}/\sigma_{700} = 2.8 \text{ where the rule holds.}$$

Table 6 gives standard vector deviation of wind for representative seasonal months for temperate, sub-tropical and tropical places:

TABLE 6. STANDARD VECTOR DEVIATION (KT.)

Station	LARKHILL				MALTA				HABBANIYA			
Months	Wi.	Sp.	Su.	Au.	Jan.	Apr.	July	Oct.	Jan.	Apr.	July	Oct.
700 mb.	32	26	21	24	21	18	16	17	15	17	14	14
500 mb.	42	34	29	34	31	26	18	24	29	26	16	15
300 mb.	56	49	45	49	48	36	22	34	40	34	19	25

Station	BAHREIN				ADEN				NAIROBI			
Months	Jan.	Apr.	July	Oct.	Jan.	Apr.	July	Oct.	Jan.	Apr.	July	Oct.
700 mb.	16	15	15	13	13	9	9	8	10	8	11	8
500 mb.	23	20	13	14	14	11	10	10	11	7	13	10
300 mb.	36	30	14	20	20	18	8	13	16	11	16	13
200 mb.	33	38	14	21	27	19	10	16	20	15	24	17

Authorities and periods for data - Larkhill, 1948-50: London, Meteorological Office, M.O.555, Parts 1, 2 and 3. Upper Air Data. Values for other stations were computed from manuscript data for the period 1950-52, available in the Upper Air Branch of the Meteorological Office.

/Considering

Considering the validity of the rule up to 300 mb., it will be seen that it fails for Malta and Habbaniya in July; for Bahrein and Aden in July and October; and that it does not work very well in any month for Nairobi.

Considering the validity of the rule up to 200 mb., Bahrein and Aden ought to provide a test in July, and Nairobi throughout the year. The rule fails in each case, although at Nairobi the rule would provide a good estimate of the 200 mb. values given those at 300 mb.

It would appear from Table 6 that in tropical air the standard vector deviation of wind increases more slowly with height than would be indicated by the rule.

From equations (6) and (14) we derive

$$\sigma/\sigma_o = s/s_o \qquad (20)$$

This is certainly true in extra-tropical regions, since equation (6) is valid; but it seems to hold good in the tropics, where in general s varies slowly with height and σ does also.

The slow variation of s with height in the tropics might be expected from equation (13)

$$s^2 = 0.72\, s''^2 + 0.6\, s'' s_o + s_o^2$$

since s'' is proportional to the standard deviation of temperature, which becomes very small in the tropics.

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Schumann, T.E.W. and van Rooy, M.P.	1951	Union of South Africa, Weather Bureau.
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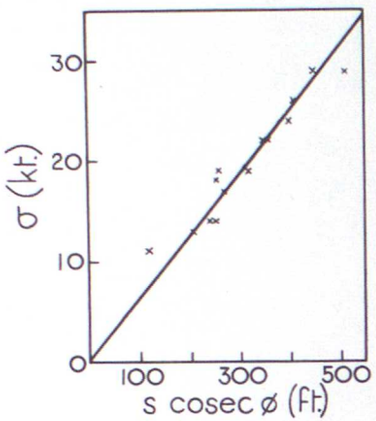
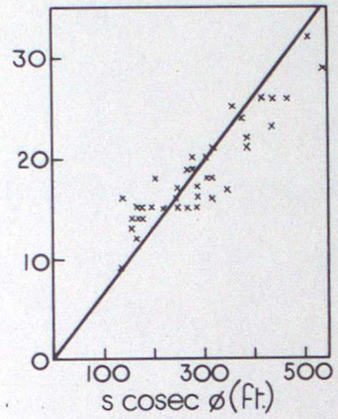
Tippett, L.H.C.	1925	Biometrika, 17, p.364.

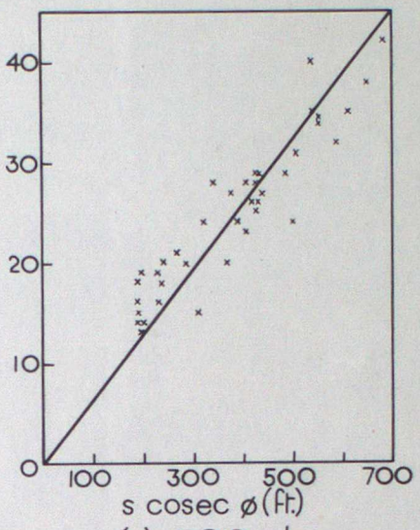
Graphs to show the relation between
s cosec ø and σ at various levels



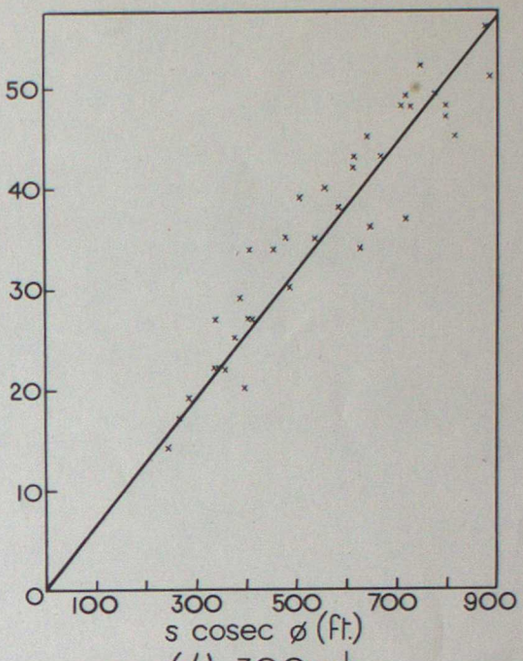
(a) 850 mb.



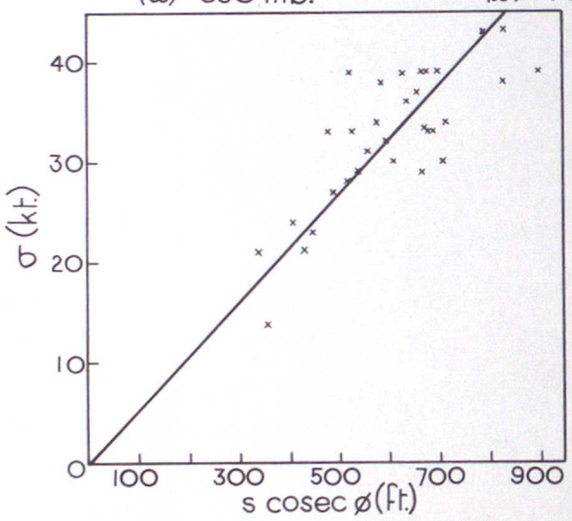
(b) 700 mb.



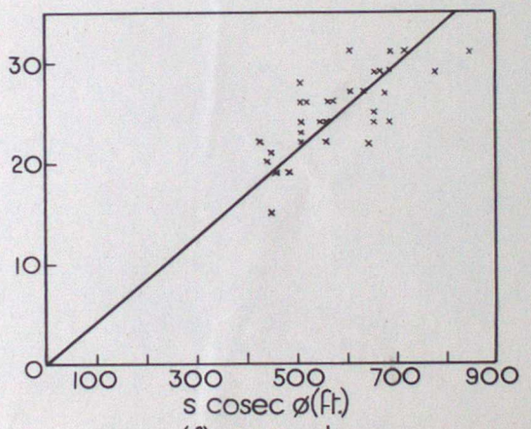
(c) 500 mb.



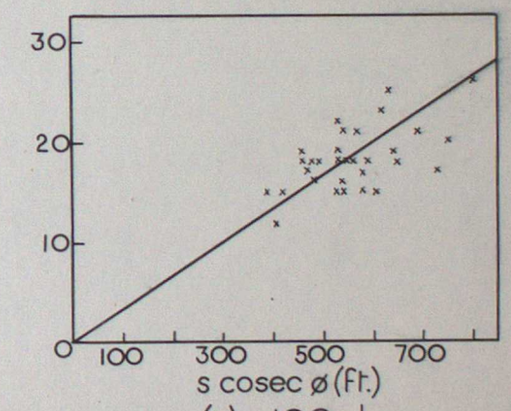
(d) 300 mb.



(e) 200 mb.



(f) 150 mb.



(g) 100 mb.

FIG. 1 (a, b, c, d, e, f, g)

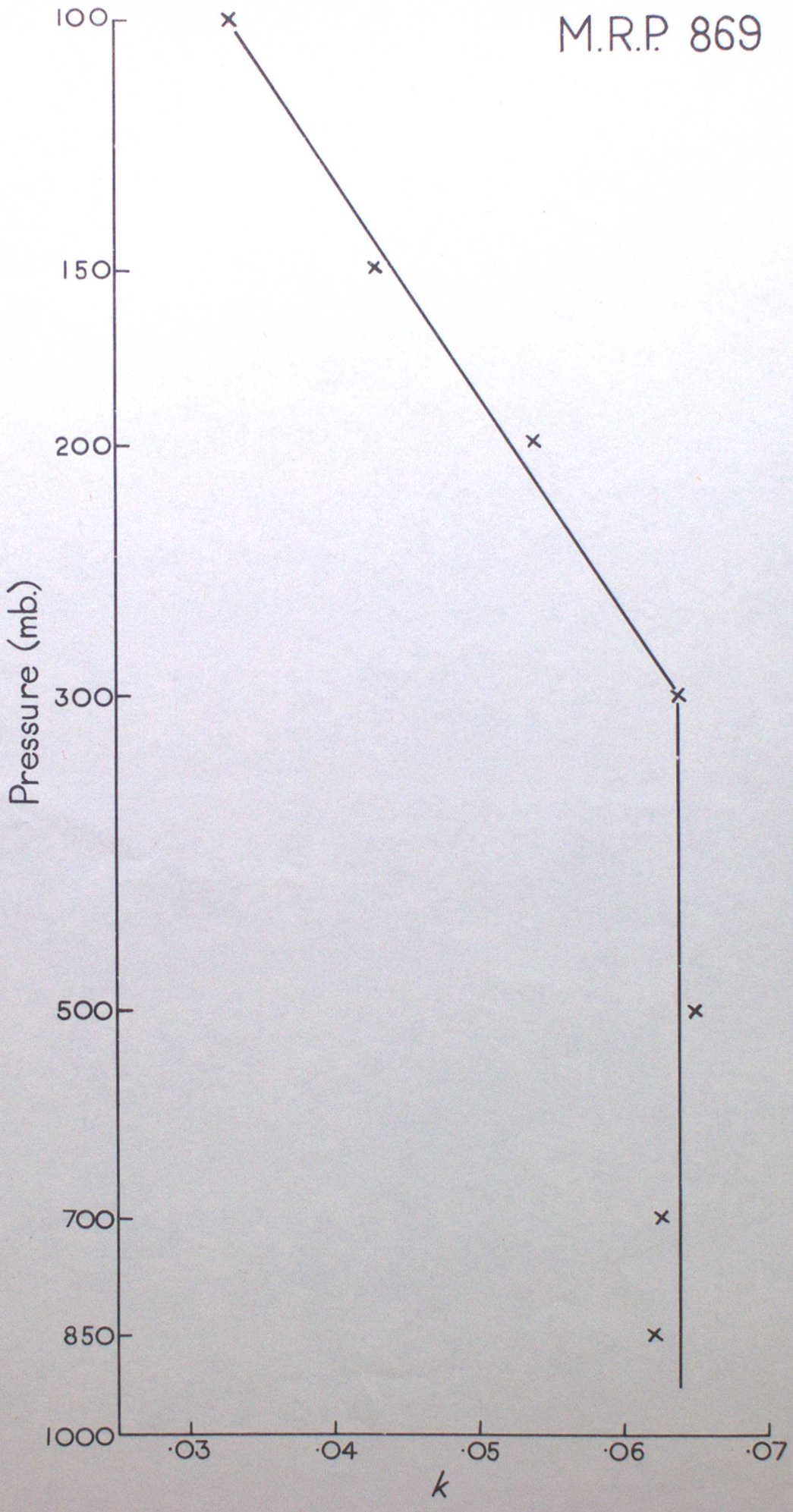


Fig. 2 Variation of k with height

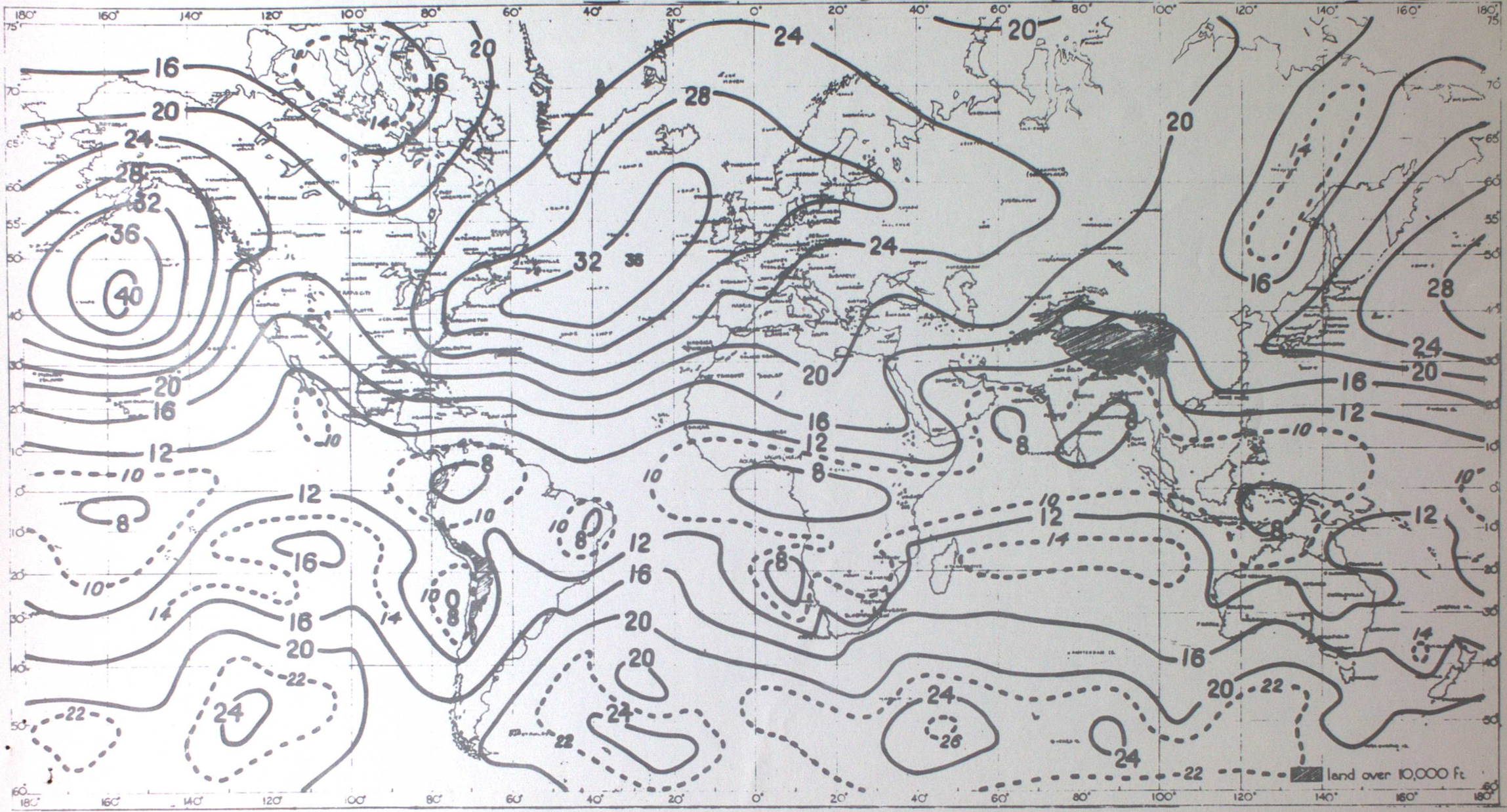


FIG. 3 ESTIMATED STANDARD VECTOR DEVIATION OF THE WIND ABOVE THE FRICTION LAYER (KT.) — JANUARY

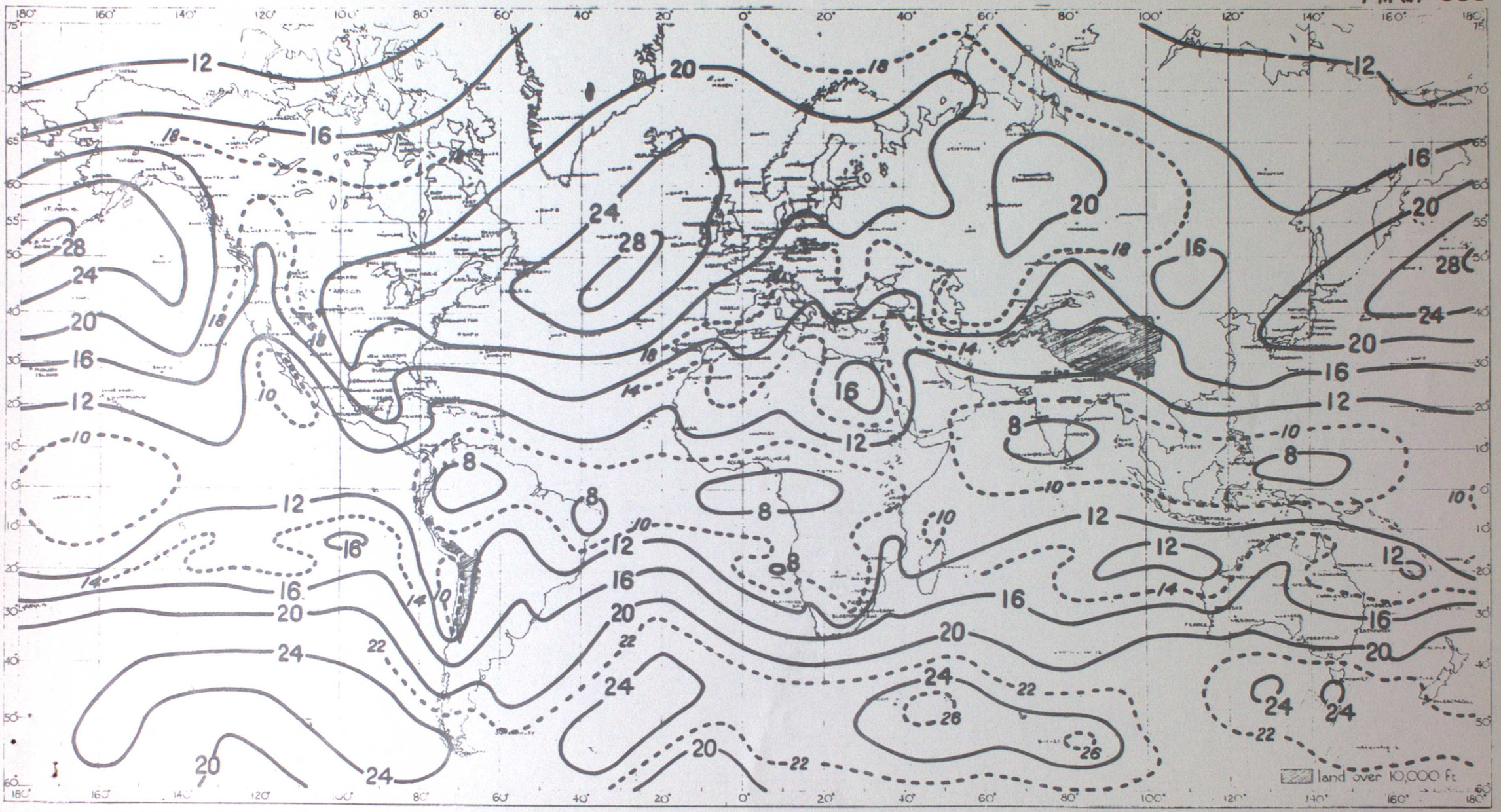


FIG. 4 ESTIMATED STANDARD VECTOR DEVIATION OF THE WIND ABOVE THE FRICTION LAYER (KT.) – APRIL

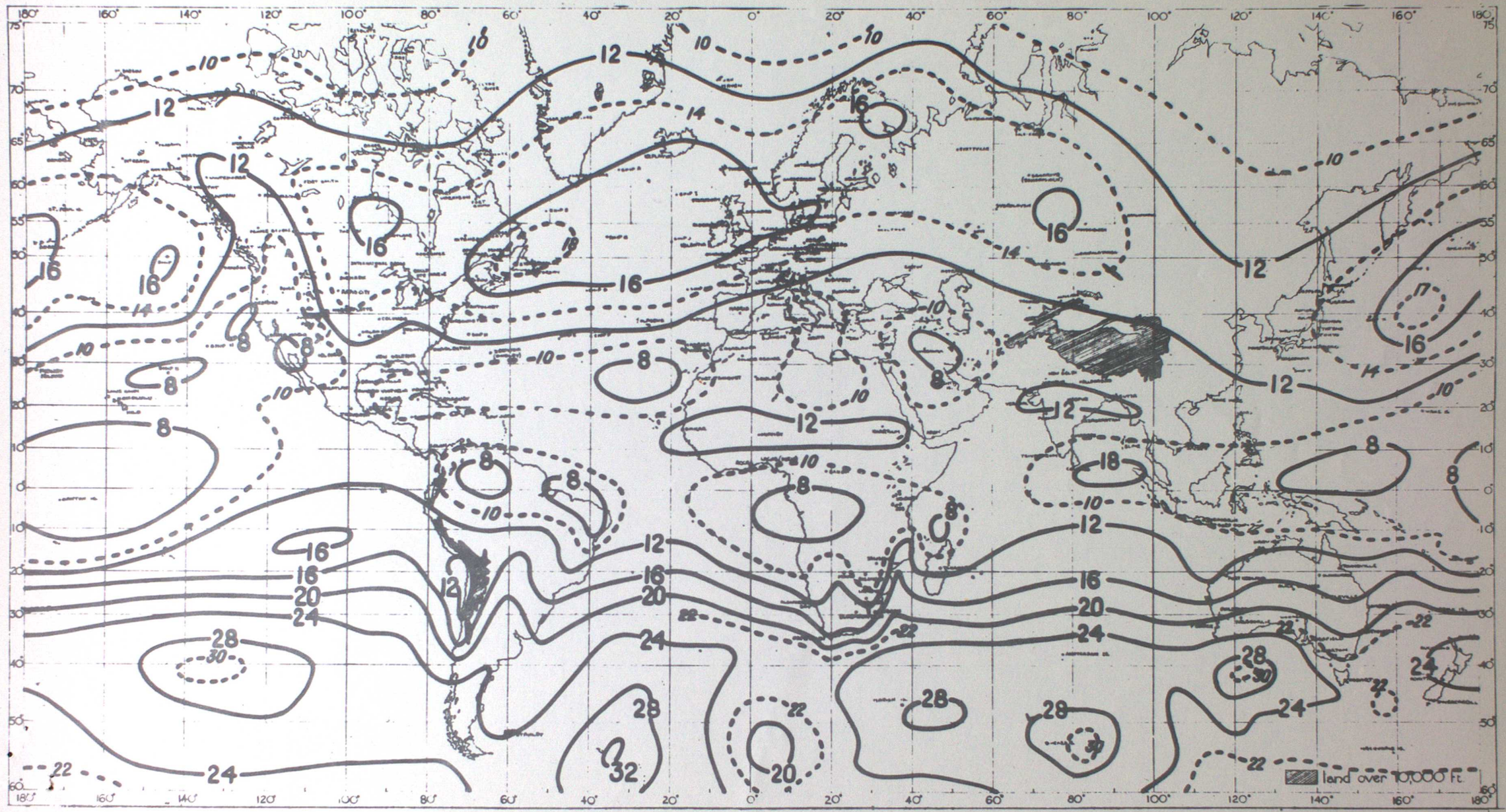


FIG. 5 ESTIMATED STANDARD VECTOR DEVIATION OF THE WIND ABOVE THE FRICTION LAYER (KT.) — JULY

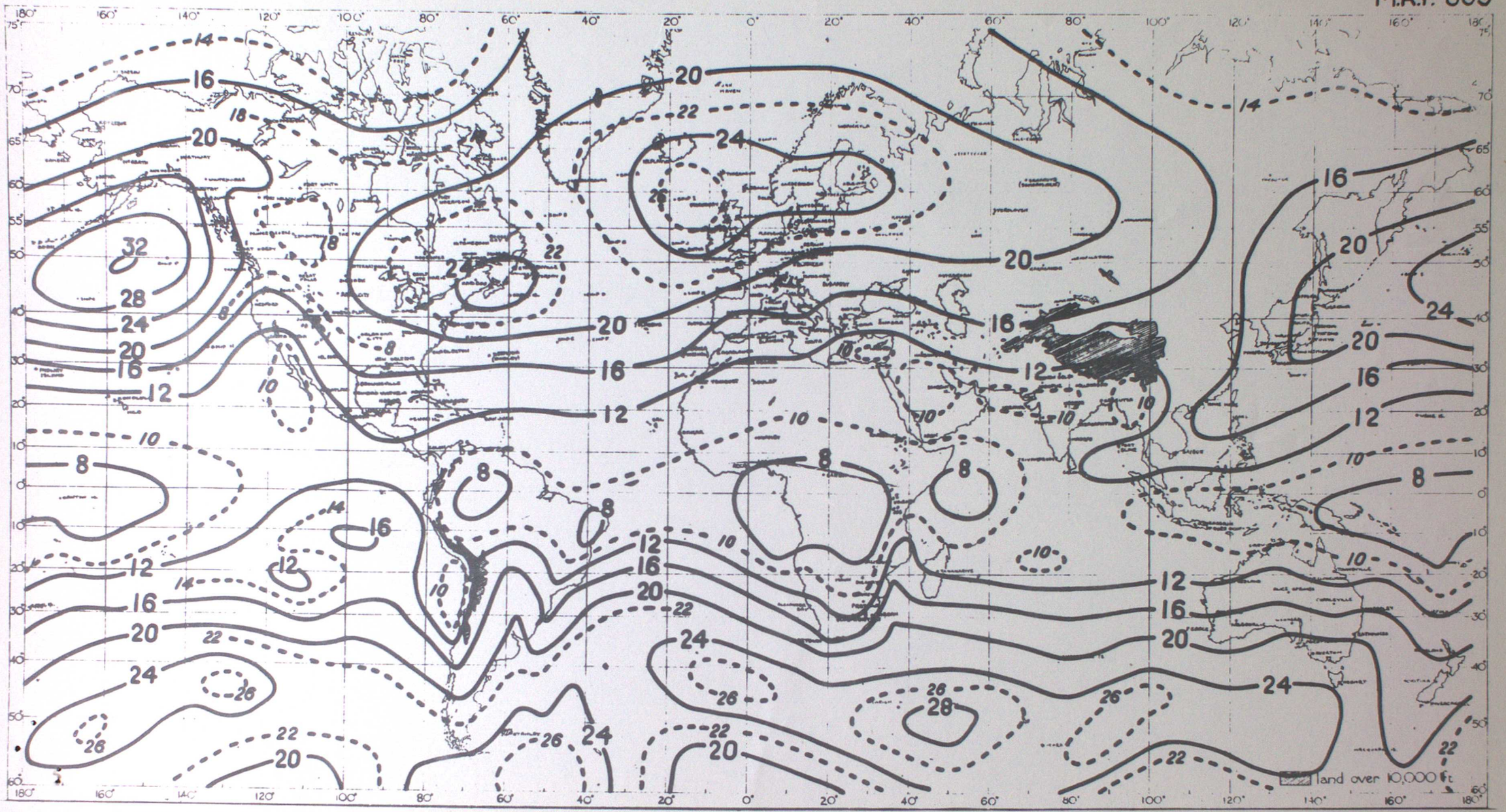


FIG. 6 ESTIMATED STANDARD VECTOR DEVIATION OF THE WIND ABOVE THE FRICTION LAYER (KT.) – OCTOBER

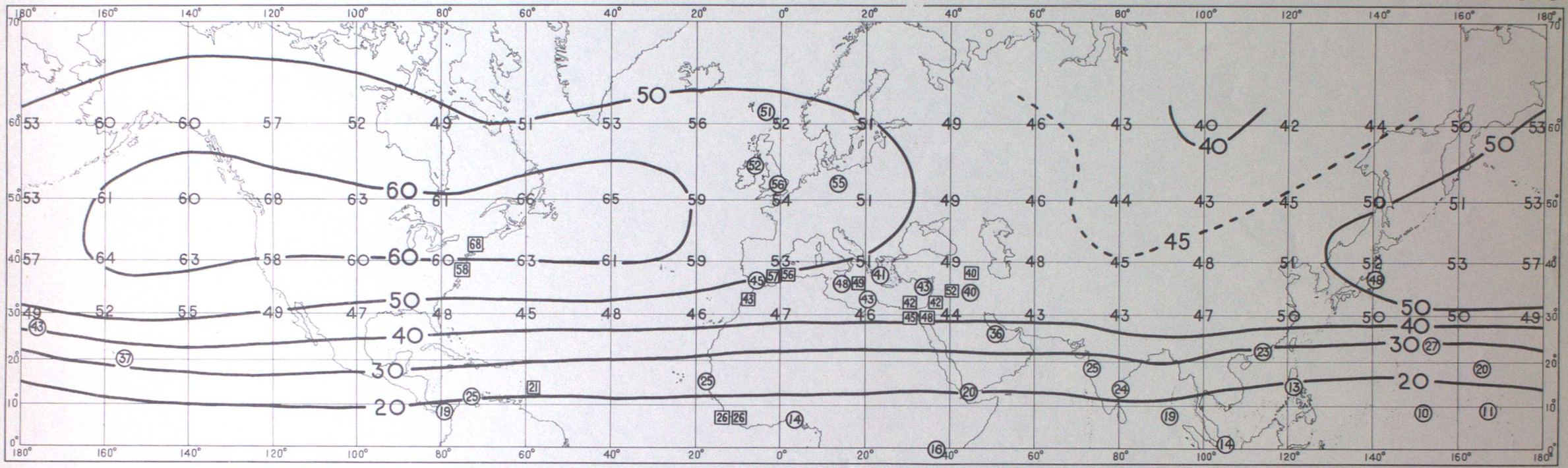
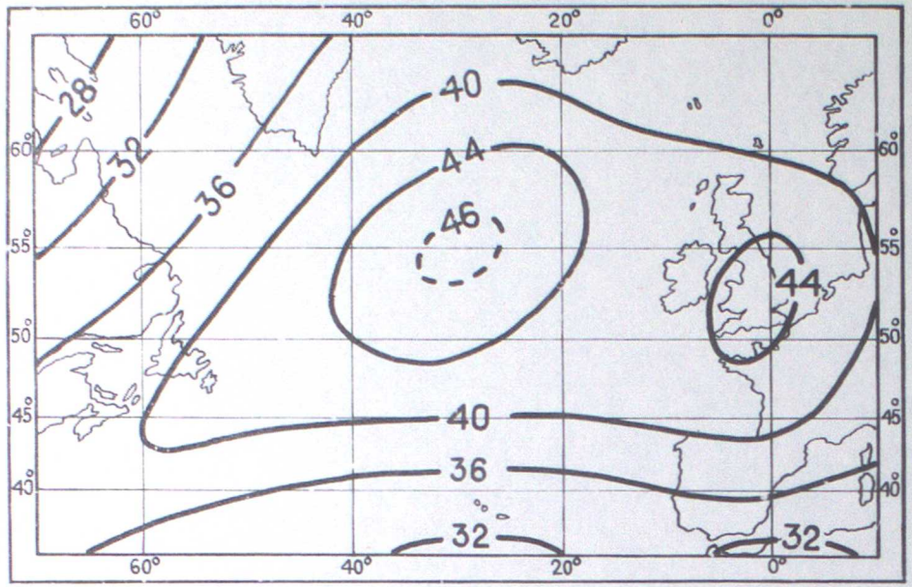
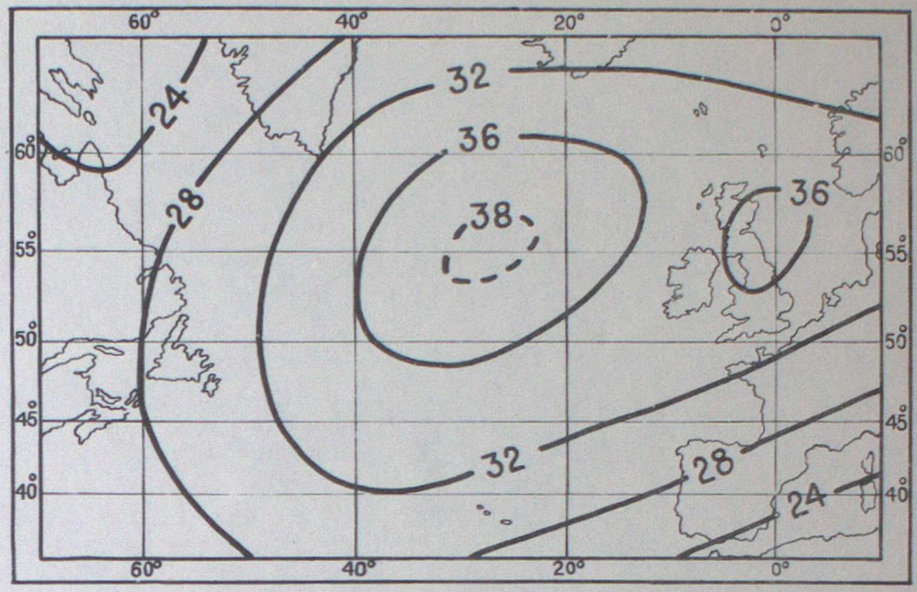


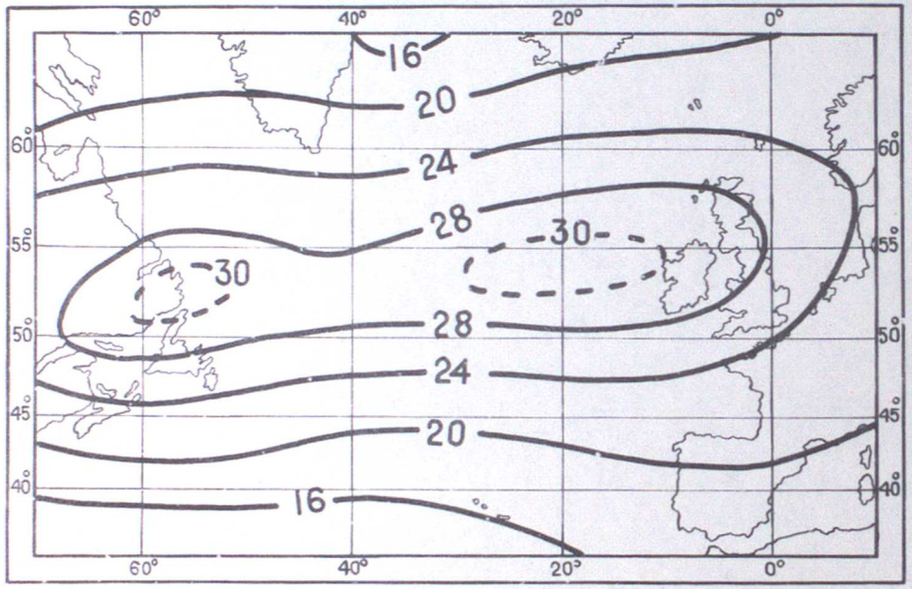
Fig. 7 Estimated standard vector deviation (kt.) at 300 mb. — January

JANUARY



APRIL





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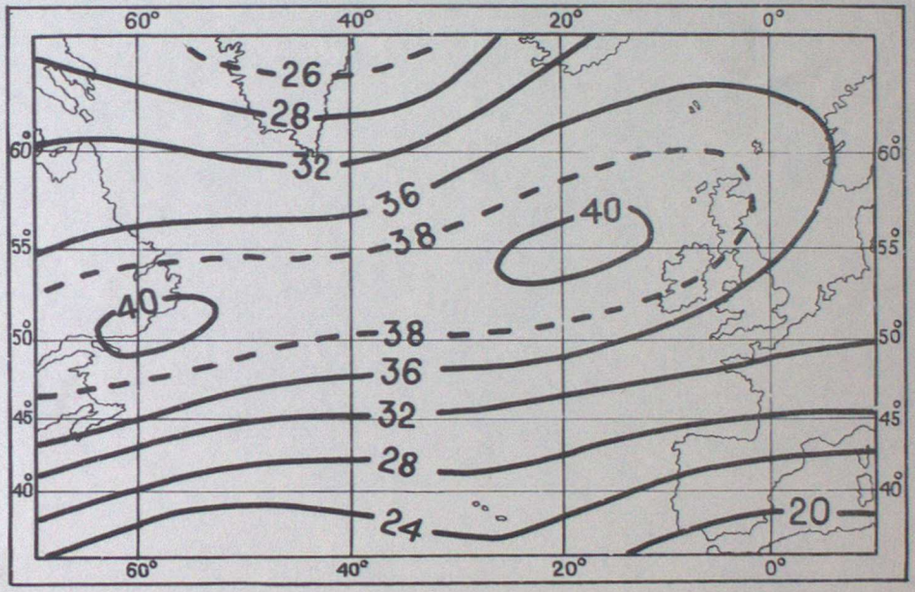

OCTOBER

Fig. 8 Estimated standard vector deviation of wind (kt.) at 500 mb.